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ABSTRACT

As a learning theory, constructivism describes knowledge as being in flux--where an individual internally constructs knowledge through social and cultural mediation. Constructivist learning theorists contend that social activity and discourse must play important roles in order for understanding to occur. This paper describes how the social constructivist view of learning can be useful in the teaching of high school algebra and in the preparation of mathematics teachers. A brief summary of the social constructivist view as a philosophy of mathematics is presented, followed by a review of learning theory associated with school algebra. Implications for the preparation of pre-service mathematics teachers are discussed. (Contains 17 references.) (ASK)

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Running Head: THE ROLE OF SOCIAL CONSTRUCTIVIST PHILOSOPHY

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The Role of Social Constructivist Philosophy in the Teaching of School Algebra and in the Preparation of Mathematics Teachers

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Introduction

Constructivism is considered a driving force in mathematics education (Malone & Taylor, 1993; Steffe, & Gale, 1995). As a learning theory, it describes knowledge as being in flux, where an individual internally constructs knowledge through social and cultural mediation (Fosnot, 1996). The act of learning is considered a self-regulatory process where new information is accommodated in order to develop representations and models of reality. Constructivist learning theorists contend that social activity and discourse play important roles for understanding to occur. Thus, the classroom is viewed as a mini society-a community of learners engaged in activity, discourse, and reflection. The teacher provides concrete and contextually meaningful experiences where students are permitted to raise their own questions, construct models, concepts, and strategies (Fosnot, 1996).

Cobb (1996) has identified a trend toward socially and culturally based activity. This is a move away from the importance of the individual in learning toward a notion that learning is influenced by cultural practices, such as completing worksheets, shopping, or packing crates in a dairy. Ernest (1998) has recently developed an epistemological view of social constuctivism. The purpose of this paper is to describe how the social constructivist view of learning can be useful in the teaching of high school algebra and in the preparation of mathematics teachers. A brief summary of the social constructivist view as a philosophy of mathematics will be discussed followed by a review of learning theory associated with school algebra. The paper concludes with implications for the preparation of pre-service mathematics teachers.

Summary of Social Constructivism

Mathematics has a long standing tradition that mathematical knowledge, associated with logic, is an infallible body of knowledge. However, Social Constructivists refute the claim that mathematics is an infallible body of knowledge. Ernest (1998) contends that the traditional views of mathematics have not ensured the infallibility of mathematical knowledge. It is evident that mathematical knowledge has evolved over time and is a human endeavor. The evolution of

mathematical knowledge is a process with historical origins and social context that play important roles. Constuctivism, in general, sees the classroom as a "minisociety" (Fosnot, 1996). Similarly on a broader scale, Social Constructivists view mathematical knowledge as being influenced by human activity and grows out of a community composed of individual mathematicians. As Ernest (1998) states, "All knowledge is rooted in basic human knowledge and is thus connected by a shared foundation . . . human agreement is the ultimate arbiter of what counts as justified knowledge" (p.48). Hence, Social Constructivism takes into account learning communities where individuals are engaged in discourse, discuss new theories, make revisions, and agree upon the new resulting mathematical knowledge.

Social Constructivism is more than a learning theory. It has become a philosophy to guide a learning theory (Ernest, 1998). Social Constructivism, as a philosophy of mathematics, must meet several criteria, one of which relates to mathematics education—how mathematical knowledge is handed down from one generation to the next, and the criterion regards the learning of mathematics as central and unavoidable for a philosophy of mathematics. This criterion is concerned with what mathematics is learned and the dialectical relationship between individuals and existing knowledge. Through schooling, an individual obtains mathematical tools and extends experiences. Opportunities are provided so that one is better able to understand one's ability to establish a personal world of mathematical objects. Another criterion related to this discussion is that a philosophy of mathematics should account for the activities of mathematicians both past and present (Ernest, 1998). This suggests that learners of mathematics should be aware of the history of mathematics and perhaps it should occupy a more prominent position in the curriculum.

Since the classroom is viewed as a "micro-world" of mathematicians that reflects the actions of the larger community of mathematicians, classroom teaching should acknowledge the belief that mathematical knowledge is socially constructed and validated (Neyland, 1995). In Cobb's (1996) view, mathematical learning not only involves individuals actively constructing knowledge but also requires an enculturation into the mathematical practices of the wider society of mathematicians. This implies that students should be empowered to make contributions to the

reconstruction of society. Students then must be encouraged to form new understanding of mathematics using interpretations of existing as an individual's reference frame (Neyland, 1995).

Mathematical knowledge is founded on the process of creating proofs, which social constructivists consider as the development of mathematical knowledge. Proofs result from a socially agreed upon set of rules and mathematical objects, such as definitions, axioms, and theorems. They are the heart and soul of mathematical knowledge. As a philosophy of mathematics, Ernest (1998) strongly refutes the Absolutist views—as a body of knowledge, mathematics is separate and apart from human activity, and mathematical knowledge and logic are valid and infallible. However, mathematical knowledge and logic are fallible since mathematical proofs are the result of social discourse within the mathematical community.

Central to the proof discourse is language. Language in mathematics has certain features such as participatory rules, behavioral patterns, and linguistic usage from which meaning is established through social patterns. Although rules and form of the game may change, the rules are learned through participation in the game. In the mathematical community, agreements, regarding the rules of mathematical proof and establishing theories, arise from a shared language with common understandings of the language and its meanings. This suggests that mathematicians have established guidelines for the verification of proofs. Depending on the social conditions, the guidelines may change as a result of a new approach to mathematical proofs, such as a proof performed by a computer. In the past, this would have been unacceptable, but the community has recognized the power of computer generated proofs; thus, the established rigor for proofs change due to particular advances. Thus, what is required for mathematical knowledge in the form of proofs change as a result of societal conditions and agreement through discourse among the community members (Ernest, 1998).

Social constructivists view language as very important because knowledge grows through language and its shared meanings. According to Ernest (1998), ". . .the social institution of language. . . justifies and necessitates the admission of the social into philosophy at some point or other" (p. 131). Mathematics has many conventions, which are basically social agreements on a

definition, assumption, or rule. Objective knowledge rests with the social framework due to linguistic rules and practices. For social constructivism, mathematical knowledge and logic are epistemologically social phenomena that include language, negotiation, conversation and group acceptance. Hence, mathematical knowledge is mathematics taken to be what is accepted and warranted by the mathematical community.

The language game requires both informal and formal mathematical practices. As in any language, the syntactic rules are important for understanding to occur among the participants. The form of the rules and their acceptance evolve within context-related linguistic and social practices. Mathematical knowledge is created in the minds of individual mathematicians, participating in language games, each of whom construct their own meaning, "mathematics is constructed by the mathematician and is not a preexisting reality that is discovered" (Ernest, 1998, p. 75). Inventions in mathematics are brought about through mathematical propositions, theorems, mathematical concepts, and the form of mathematical expression. The social nature of mathematics is established by meaning derived from the context-related linguistic and social practices. Consequently, mathematical knowledge is founded on human persuasion and acceptance.

Proofs develop within the social community of mathematicians and are considered mathematical knowledge when fully established through a testing process. The social nature of mathematics is seen when a proof is presented to a body of mathematicians and placed under scrutiny; it is faced with either acceptance or rejection, if flaws are present in the proof, a new and improved version is then presented. This cycle continues until there is agreement. Mathematical knowledge is consequently, tentative, continually being tested and not necessarily established by the rigors of proof because assumptions are agreed upon tenets that may change. Conjectures, proofs, and theories arise from a community enterprise that, in the philosophy of mathematics, includes both informal mathematics and the history of mathematics (Ernest, 1998). The National Council of Teachers of Mathematics has recognized the power of reasoning and proof in mathematics classrooms. The writers of NCTM's (1998) draft of the Principles and Standards of School Mathematics suggest that all students should be able to "recognize reasoning and proof as

essential and powerful parts of mathematics, make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proofs, and select and use various types of reasoning and methods of proof as appropriate" (p. 248). Consequently, it appears that the draft standards as a social constructivst position, recognizing the importance of socially evolved proofs.

In sum, the transmission of mathematical knowledge is a very important aspect of the social constructivist philosophy. As a micro-community, mathematics exists in a social context; learners knowledge of mathematics is related to shared social activities. The function of mathematical symbol systems as semiotic tools must be socially acquired and mastered for obtaining mathematical knowledge and competence (Ernest, 1998). Semiotic tools have their own rhetorical features and become part of the overall social make-up and context of the classroom. The acquisition and use of mathematical concepts are not readily detached from social activities and are often elicited as a result of an engagement with a specific social situation. The mathematics classroom also has its language games and social forms of life shared by a community (Ernest, 1998).

School Algebra

Algebra, as a body of knowledge, does have historical origins. Algebra, as a symbol system with transformational rules, can be viewed as a language with historical roots. For example, a distinction between using letters to represent unknowns in equation solving and using letters to represent givens in expressing general rules governing numerical relations can be made. The historical background of algebra establishes a foundation for the development of algebra as a symbol system, moving from a procedural to a structural perspective. Kieran (1991) believes that some of the cognitive processes involved in learning school algebra follow the historical development of algebra as a system.

Three stages are identified by Kieran (199). The first stage is the rhetorical stage, prior to Diophantus, which is characterized by the use of ordinary language descriptions for solving particular types of problems. There was little use of symbols or special signs to represent

unknowns. The second stage is called syncopated algebra and was started by Diophantus who introduce letters to represent unknown quantities. During the third to sixth centuries, algebraists were concerned with discovering the identity of the letters rather than express them in the general sense. Vieta's use of letters to stand for a given as well as an unknown quantity was the beginning of the third stage. It was now possible to express general solutions and to use algebra as a tool for proving rules governing numerical relations. Consequently, this allowed for the creation of new concepts like the function concept. The function concept evolved from Eulers view that represented a procedural notion of an input-output process to Dirichlet's structural conception as an a correspondence between real numbers, and then the function concept was generalized by Bourbaki as a relation between two sets (Keiran, 1991).

Clearly, it can be seen that much of school algebra follows this historical development. Kieran (1991) defines algebra "as the branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures and with operating on those structures" (p. 391). This implies that school algebra has both procedural and structural aspects. Procedural refers to arithmetic operations, such as evaluating the expression $3x+y$, where $x = 3$ and $y = 2$, the result is 11. A second example is solving an equation like $2x + 3 = 7$ by substituting various values for x. The objects that are operated on are the numeric instantiations rather than the algebraic expressions (Keiran, 1991). The structural aspects include topics like simplifying and factoring expressions solving equations by performing the same operation on both sides and manipulating functional equations. Structural aspects refer to operations on algebraic expressions rather than on numbers, such as combining like terms in the expression $3x + 2y + x$ which simplifies to $4x + 2y$ or $2(2x+y)$. Hence, algebra with a procedural and structural foundation mirrors a language, where mathematical objects must be given meaning.

School algebra for most students is their first encounter with a symbol system that can use arithmetic processes. In order to operate on these abstract algebraic expressions, teachers must present these expressions as mathematical objects with meaning. Both pre-service and in-service teachers must come to understand that learning algebra does have a social dimension.

Just as a language has semantics and syntax, so too does algebra. Letters are used in algebra not for words but also for representing mathematical objects. School algebra is symbol system with a syntax that allows particular conventions to be used for manipulating terms and simplifying expressions. The ability to understand the rules associated with a language game is very important. Teachers should strive for increasing understanding of what the algebraic statements represent and why a particular manipulation is a correct one to employ. A thorough understanding of the structural aspects of mathematical properties is necessary--the semantics of algebraic expressions. Semantic problems occur as a result of a poor understanding of the relations and mathematical structures that underlie algebraic symbols, and syntactic difficulties arise from the introduction and manipulation of the symbols in algebra (Booth, 1989). Therefore, it is important that pre-service teachers of mathematics gain a knowledge of how to improve both semantic and syntactic understanding of algebraic concepts.

The language of algebra with its semantics and syntax must be properly presented in order for conceptual understanding to occur. The language rules of algebra can be reinforced through the teaching of concepts in conjunction with semantic and syntactic meaning. Sfard (1991) identifies three phases of conceptual development in school algebra. The first phase is interiorization where students perform some process on already familiar mathematical objects. The second phase is called condensation. In this phase, the operation or process is reformulated into a more manageable unit, until the new object is conceived procedurally. The third phase is refection where there is a sudden ability to see something familiar in a new light. The new object is thus detached from the process that produced it.

Algebraic concepts cannot be taught in isolation. Pre-service teachers should experience each of the three phases. The first phase, interiorization, appears to require that teachers recognize what students know about various mathematical objects. Pre-service teachers then must be aware of techniques to determine where their students are in relation to mathematical objects. This may be accomplished through modeling discourse in mathematics or mathematics education classes. Pre-service teachers should recognize how condensation is beneficial. This phase seems to require

extensive communicative practices, such as explaining their understanding of a problem situation or symbolic representations of a problem. They should experience the connection between the semantic aspect of a problem representation and its syntactic procedures. Clearly, the third phase is the use of a concept to a new situation. In the past, this was referred to as transfer.

Social Constructivism-Implications for School Algebra Teaching

Clearly, school algebra has a sociocultural perspective, and school algebra requires mutual appropriation, between the student and the teacher, of each other's contributions. Pre-service teachers must experience the mediating process so that they may be better prepared to appropriate their students' personal meanings and culturally established mathematical meanings. Pre-service teachers must come to understand the value of promoting an active algebra classroom culture that models or reflects practices of the wider mathematical community.

Social constructivism is based on an fallibist epistemology that regards knowledge as lived and socially accepted (Ernest, 1996). A connection between abstract algebra and school algebra should be made (Confery, 1993). Pre-service algebra teachers should be prepared to use various features of abstract algebra, such as choosing proofs that are appropriate for school algebra. The proofs should be developed in such a way as to reflect the negotiation process that occurs among mathematicians. Pre-service teachers ought to be prepared to offer their students the opportunity to write or discover proofs of school algebraic concepts, such as the quadratic formula, or the basic ideas related to groups, fields, and rings. The process would involve both semantic and syntactic skills for enhancing their understanding of the mathematical structure associated with school algebra. Public conjecturing leads to a discourse where students challenge ideas by offering counter examples, explain the counter examples, shifts in definitions results that lead to new conjectures and tentative proofs (Confrey, 1993).

Pre-service teachers should also have a repertoire of historical aspects of algebra. This will help their students to understand that algebra has roots and that algebra is not a subject to study because certain adults require it. Algebra has a history with a long development and interesting

social interactions, such as the proving of the cubic equation, Descarte's application of geometry to algebra creating the coordinate system. This offers an opportunity for school algebra students to see that various cultures played roles in the development of algebra.

Example of a Social Constructivsit Activity

Activity theory was proposed by Leont'ev (1981) who contends that knowledge is constructed as a result of personal, subjective experience of an activity with a motive or goal. The goal or expectation may be reached by different actions. There are two levels associated with activity theory, one a personal plane and two, an intellectual plane. Semenov (1978) described the personal plane as the place where cognitive processes are involved in the monitoring and evaluating the ongoing problem solving effort. This requires a conscious awareness of the goal and self-regulatory processes that occur during reflection about objective information and problem solving strategies. The problem is internalized into the personal plane, becoming a part of the student's consciousness. The intellectual plane refers to the development of the content of the problem. It appears that through activity theory school algebra students will have an opportunity for discourse to increase semantic and syntactic features of a problem. Crawford (1996) concluded that "shared meanings are created thorough a shared history of action" (p. 138).

The following is an example of an activity that reflects social constructivist aspects for learning school algebra. The example presented below uses cooperative learning, discourse, language skills for explaining mathematical language, and negotiation of meanings in the context of mathematical modeling. The activity addresses another of Ernest's (1998) criterion for judging the adequacy of a philosophy of mathematics--applications of mathematics. Mathematical modeling is a tool that can be used for extending the boundary of the philosophy of mathematics to include mathematics and its relations with other human knowledge. Social Constructivism views mathematics and other areas of knowledge as interconnected with persons and various knowledge contexts. A web is created that "reflects the origin of mathematics as a language to describe, predict, and regulate quantitative and spatial phenomena" (Ernest, 1998, p. 263). Mathematical

modeling can be viewed as a mathematical language that describes certain aspects of empirical and social realities.

Pre-service teachers should have an opportunity to experience mathematics in a similar fashion so that, with this experience, they may be better able to employ a similar method. There are several stages that occur in this process of using mathematical language to describe real-world phenomena, but the stages are not necessarily linear. These include:

- Identify data from a real-life situations and be presented in a verbal representation.
- Brainstorm and formulate mathematical questions defining the problems that are associated with the situation. Here language is used to identify and understand the dimensions of the concrete representation.
- Data is organized to summarize the data, identify and define variables, constraints, and goal(s) for the identified problem(s).
- Mathematical representations and language are used to prepare data for algebraic explorations.
- Formulate hypotheses about the nature of the future solutions.
- Develop a visual model (graphic) of the information to verify the hypotheses and produce algebraic or graphical solutions to the problem.
- Use language to formulate in written terms the solutions produced, decisions made, the predictions created and the conclusions attained by the investigative analysis.

Problems may be identified from local resources, such as newspapers or magazines. An issue related to students or the community in particular can be found in a local paper. For example, one issue in a local setting may be to determine whether or not it is necessary to release fresh water into a bay. A mock research facility may be established in the classroom where they assume roles of mathematicians and/or scientists. There may be an elaborate skit performed that sets the stage to begin investigations. Teams of scientists consisting of students may be formed as independent entities that may investigate different but related issues concerning the situation. On team may be interested in the relationship between the amount of inflow to the weight of landed fish. Another

may be interested in how salinity effects the weight of landed fish. Data can be collected for several species of fish and shrimp and the amount of released fresh water. The data may be modeled mathematically using graphing technology. Discussions are conducted about the issue, the appropriateness of a mathematical model. Findings and recommendations may be presented to the class and/or the wider community.

The Preparation of Mathematics Teachers

Social constructivism incorporates the historical dimension of one's personal growth in mathematical knowledge, paralleling the growth of the discipline (Ernest, 1998). The role of social constructivism in mathematics education should begin with an examination and reconstruction of prior beliefs and experiences (Northfield, Gunstone, & Erickson, 1996) to arrive at the realization that teaching itself is a social construction, ideas for how to teach and what to teach are developed through social norms. A situational problem solving approach to teaching mathematics exposes pre-service teachers to an alternative view. Projects allow teachers to accommodate diverse interests and abilities. Cognitive structures are shaped through social influences using language, an interactive approach to creating problems and determining the viability of solutions (Taylor, 1993). Teacher preparation programs using a similar model should encourage an essential capability, close listening. Confrey (1993) considers close listening as an ability to imagine what the student is seeing mathematically, to request from the student explanations and elaboration of the problem that is being addressed, how the problem is being solved, and to elicit self-monitoring strategies. This cannot be accomplished without a deep and flexible knowledge of subject matter and pedagogical strategies. "Teaching teachers to cultivate student invention, to make use of opportunities when they occur and to challenge and revise their own views of subject matter is a critical concern" (Confrey, 1993, p. 307).

Pre-service mathematics teachers should be presented with actual examples of classroom events and have opportunities to work with their future clients, analyze segments of videos and practice in micro-teaching settings. They should be supervised in a manner that requires

observation and critique of their pedagogical strategies and content knowledge from a social constructivist perspective. Another capability to develop while in a program is the knack for using tasks and materials in contextual situations that foster the need to learn algebra through activities that require embedding ideas in student oriented challenges and creating a classroom environment that supports and encourages debate, exchange, and negotiation of ideas (Duit, & Confrey, 1996).

Conclusion

A social constructivst approach to teaching school algebra necessitates changes in how mathematics teachers are prepared. Social Constructivism as a philosophy of mathematics has the potential for providing a foundation for the teaching of algebra. Since mathematics is a language, and especially school algebra as a symbol system, it follows that an approach to teaching school algebra should include a historical background of concepts, discourse for establishing meaning for symbols and the use of mathematical language for describing social phenomena, and in verifying proofs develop within the micro-community of the classroom. In many college mathematics programs, students have little opportunity to experience discourse related to mathematics or public verification of proofs. Algebra, as a language, has its semantic and syntactic meanings. Future school algebra teachers should be adept at developing both meanings during instruction. Social constructivism recognizes the role of informal knowledge in the genesis of mathematical knowledge. Likewise, school algebra students' informal knowledge should be used to develop a formal knowledge of the semantics and syntax of algebra. They should develop the capabilities for providing historical backgrounds of particular concepts so that their students can gain a sense that mathematics is a human creation, and abstract algebra and proofs should have a larger role in the teaching of school algebra. As a result, middle and high school algebra students, will gain a thorough understanding of the relationship among the concepts of algebra, its history, and the evolution of mathematical knowledge. Teachers would also obtain from various sources, such as historical perspective, and the use of language and dialogue to enhance the learning of algebraic concepts.

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